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SOME PROBLEMS OF HEAT TRANSFER IN LIQUID-COOLED REACTORS

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1. INTRODUCTION

The modern nuclear power reactors are highly forced. Large heat fluxes and specific power, rigid limitations on the coolant and fuel elements temperatures demand knowledge of temperature distribution in the core with a high reliability.

The main problem in thermal reactor design is to prove reliably that the impermissible coolant and fuel element temperatures will not be realized at any reactor operational regimes.

Specific features of heat transfer problems in modern reactor technology are defined with core arrangement, which for the most of power liquid cooled reactors is a set of assemblies of canned cylindrical fuel elements. The knowledge of the flow pattern and heat transfer rate is necessary for determination of the temperature distribution in such a complicated geometry. In compact cores it needs to account for the effect of the axial heat flux variations on the heat transfer coefficient. These problems are considered in the paper.

2. TRANSPORT PROCESSES IN TURBULENT FLOW

The motion of fluids in nuclear reactors is turbulent. Theoretical study of the processes occurring in turbulent flows is very complicated. Due to irregularity of turbulent motions it is desirable to applicate statistical methods. However, the realization of such an approach in all details is very difficult. Therefore, in practice semi-empirical phenomenological theories are used. The object of these theories is to find some relations between mean and fluctuating parts of the motion. On Prandtl's mixing-length theory the velocity pulsations in a flat channel can be presented as

$$u^1 \sim l_y \frac{dU}{dy},$$

where l_y -- is a characteristic distance passed by the pulsations.

Let us derive a more exact formulation of Prandtl's hypothesis. Introducing the suitable Green functions, we can write the equations of motion for the turbulent fluctuations

$$\frac{\partial u_i^1}{\partial t} + U_k \frac{\partial u_i^1}{\partial x_k} + u_k^1 \frac{\partial U_i}{\partial x_k} = - \frac{1}{\rho} \frac{\partial \rho}{\partial x_i} + \nu \frac{\partial^2 U_i^1}{\partial x_k^2} + \frac{\partial}{\partial x_k} (\bar{u}_i^1 u_k^1 - u_i^1 \bar{u}_k^1) \quad (1)$$

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That Green's function is determined by the following equation

$$\frac{\partial V_0}{\partial t} + (U \nabla) V_0 - \nu \Delta V_0 = -\delta(\bar{r} - \bar{r}^1) \delta(t - t^1) \quad (2)$$

with the corresponding boundary conditions.

The function V_0 describes the velocity distribution in liquid flow under an instantaneous local disturbance.

Then equations (1) can be written as

$$u_i^1(\bar{r}, t) = \int V_0(\bar{r}, \bar{r}^1, t, t^1) \left[\frac{1}{\rho} \frac{\partial p^1}{\partial x_i} + u_k^1 \frac{\partial U_i}{\partial x_k} - \frac{\partial}{\partial x_k} (\overline{u_i^1 u_k^1} - \overline{u_i^1} \overline{u_k^1}) \right] d\bar{r}^1 dt^1 \quad (3)$$

Such a form of Eq. (1) permits an iteration method to be applied. To obtain the first approximation formulas we may neglect in Eq. (3) all nonlinear terms and the pressure pulsations.

Then

$$u_i^1(\bar{r}, t) \approx \int u_k^1 \frac{\partial U_i}{\partial x_k} V_0(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (4)$$

Provided the width of the region, where the probability V_0 is not zero, is small compared with that one, where significant change of the mean velocity occurs, Eq. (4) can be simplified.

Expanding the mean velocity gradient into a Taylor series near the point \bar{r} and limiting to two series terms, we obtain the approximate formulae for the velocity fluctuations

$$u_i^1(\bar{r}, t) \approx \frac{\partial U_i}{\partial x_k} \int u_k^1(\bar{r}^1, t^1) V_0(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 + \frac{\partial^2 U_i}{\partial x_k \partial x_e} \int |\bar{r} - \bar{r}^1| e^{u_k^1(\bar{r}^1, t^1)} V_0(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (5)$$

In the turbulent core near the flow symmetry axis the first mean velocity derivatives are small. Therefore the velocity pulsational components are determined by the second items in Eq.(5), i.e. are dependent upon the second mean velocity derivatives. Conversely, far from the flow symmetry axis the first items of the equation are large. In this region

$$u_i^1(\bar{r}, t) \approx \frac{\partial U_i}{\partial x_k} \int u_k^1(\bar{r}^1, t^1) V_0(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 - \frac{\partial U_i}{\partial x_k} \quad (6)$$

Note, that the quantity

$$l_k(\bar{r}, t) = \int u_k^1(\bar{r}^1, t^1) V_0(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1$$

has the length dimension and represents the width of the region which is a velocity pulsation "supplier".

To improve Green's function we must take into account the turbulent diffusion of the velocity fluctuations. Using these relations and determining the pulsational diffusion coefficient by means of the equality

$$D = \frac{\overline{u_i^1 u_k^1} - \overline{u_i^1} \overline{u_k^1}}{\frac{\partial u_i^1}{\partial x_k}}, \quad (7)$$

we may introduce the new Green function by means of the equation

$$\frac{\partial V}{\partial t} + (U \nabla) V - \nabla D \nabla V - \nu_A V = -\delta(\bar{r} - \bar{r}^1) \delta(t - t^1) \quad (8)$$

Then the second approximation velocity pulsation formulae are

$$u_i^1(\bar{r}, t) \approx \int \frac{\partial U_i}{\partial x_k} \cdot u_k^1(\bar{r}^1, t^1) V(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (9)$$

As above, the velocity fluctuations in the region not far from the wall equal

$$u_k^1(\bar{r}, t) \approx \frac{\partial U_i}{\partial x_k} \int u_k^1(\bar{r}^1, t^1) V(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (10)$$

Using this expression the turbulent stress tensor components can be calculated

$$\overline{u_i^1 u_j^1} = \lim_{v \rightarrow \infty} \frac{1}{2v} \frac{\partial U_i}{\partial x_k} \int_{-v}^v u_j^1(\bar{r}, t) dt \int u_k^1(\bar{r}^1, t^1) V(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (11)$$

Since Green's function is dependent on the difference $r = t - t^1$ only, relation (11) can be written in the more convenient form

$$\begin{aligned} \overline{u_i^1 u_j^1} &= \lim_{v \rightarrow \infty} \frac{\partial U_i}{\partial x_k} \int_0^v dr \frac{1}{2v} \int_{-v}^v u_j^1(\bar{r}, t) dt \int u_k^1(\bar{r}^1, t - r) V(\bar{r}, \bar{r}^1, t, r) d\bar{r}^1 = \\ &= \frac{\partial U_i}{\partial x_k} \int_0^\infty dr \int \overline{u_j^1(\bar{r}, t) u_k^1(\bar{r}^1, t - r)} V(\bar{r}, \bar{r}^1, t, r) d\bar{r}^1 = \\ &= \frac{\partial U_i}{\partial x_k} \int_0^\infty K_{jk}(\bar{r}, r) dr = \nu_{jk}^T \cdot \frac{\partial U_i}{\partial x_k} \end{aligned} \quad (12)$$

The factor $\nu_{jk}^T(\bar{r}) = \int_0^\infty K_{jk}(\bar{r}, r) dr$ is the component of the eddy diffusivity tensor. In the Lagrange coordinate system the function $v(\bar{r}_1, r)$ approximately equals $v \approx e^{-kr}$, where $k = \frac{\alpha(v + D)}{e^2}$, α is constant, e is a characteristic dimension of the region.

Then for homogeneous isotropic turbulence may be written as the eddy diffusivity of momentum

$$\nu^T = \int_0^\infty \overline{u^1(t) u^1(t-r)} e^{-kr} dr \quad (13)$$

This expression correlates with Taylor's determination of the turbulent diffusion coefficient [1, 2].

Similarly using the equations for the temperature fluctuations

$$\frac{\partial T^1}{\partial t} + U_k \frac{\partial T^1}{\partial x_k} + u_k^1 \frac{\partial T}{\partial x_k} = a \frac{\partial^2 T^1}{\partial x_k^2} + \frac{\partial}{\partial x_k} (\overline{u_k^1 T^1} - u_k^1 T^1) \quad (14)$$

and introducing temperature Green's function by means of the equation

$$\frac{\partial W_o}{\partial t} + (U \nabla) W_o - a \Delta W_o = -\delta(\bar{r} - \bar{r}^1) \delta(t - t^1) \quad (15)$$

we can obtain the integral equations for the temperature fluctuations in the following form

$$T^1(\bar{r}, t) = \int W_o(\bar{r}, \bar{r}^1, t, t^1) [u_k^1 \frac{\partial T}{\partial x_k} - \frac{\partial}{\partial x_k} (\overline{u_k^1 T^1} - u_k^1 T^1)] d\bar{r}^1 dt^1 \quad (16)$$

In the first approximation

$$T^1(\bar{r}, t) \approx \int u_k^1 \frac{\partial T}{\partial x_k} W_o(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (17)$$

or with the same assumptions, which have been used in deriving of formula (5)

$$\begin{aligned} T^1(\bar{r}, t) \approx & \frac{\partial T}{\partial x_k} \int u_k^1(\bar{r}^1, t^1) W_o(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 + \\ & + \frac{\partial^2 T}{\partial x_k \partial x_e} \int |\bar{r} - \bar{r}^1| e^{u_k^1(\bar{r}^1, t^1)} W_o(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \end{aligned}$$

In the region not far from the wall

$$T^1(\bar{r}, t) \approx \frac{\partial T}{\partial x_k} \int u_k^1(\bar{r}, t^1) W_o(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 = \frac{\partial T}{\partial x_k} \cdot l_k^T(\bar{r}, t) \quad (18)$$

where $l_k^T(\bar{r}, t)$ is an analog of the mixing-length for the temperature pulsations. Just as in equality (7) it can be approximately accounted for the turbulent diffusion of the temperature pulsations by means of introducing the turbulent diffusion coefficient

$$D^T = \frac{\overline{u_k^1 T^1} - u_k^1 T^1}{\frac{\partial T^1}{\partial x_k}} \quad (19)$$

Then new temperature Green's function is determined by the equation

$$\frac{\partial W}{\partial t} + (U V) W - V D^T V W - a \Delta W = -\delta(\bar{r} - \bar{r}^1) \delta(t - t^1) \quad (20)$$

Then the temperature fluctuations

$$T^1(\bar{r}, t) \approx \frac{\partial T}{\partial x_k} \int u_k^1(\bar{r}^1, t^1) W(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (21)$$

The components of the turbulent heat flow vector can be defined by

$$c \cdot \rho \cdot \overline{u_i^1 T^1} = \lim_{v \rightarrow \infty} \frac{c \cdot \rho}{2v} \cdot \frac{\partial T}{\partial x_k} \int_v^\infty u_i^1(\bar{r}, t) dt \int u_k^1(\bar{r}^1, t^1) W(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 dt^1 \quad (22)$$

Since W is dependent on the difference $r = t - t^1$,

$$\overline{u_i^1 T^1} = \frac{\partial T}{\partial x_k} \int_0^\infty dr \lim_{v \rightarrow \infty} \frac{1}{2v} \int_v^\infty u_i^1(\bar{r}, t) dt \int u_k^1(\bar{r}^1, t - r) W(\bar{r}, \bar{r}^1, t, t^1) d\bar{r}^1 = \frac{\partial T}{\partial x_k} \int_0^\infty K_{ik}^T(r) dr = a_{ik}^T \frac{\partial T}{\partial x_k} \quad (23)$$

where

$$K_{ik}^T(\bar{r}, r) = \overline{\int u_i^1(\bar{r}, t) u_k^1(\bar{r}^1, t - r) W(\bar{r}, \bar{r}^1, t, r) d\bar{r}^1}$$

The quantity $a^T(\bar{r}) = \int_0^\infty K^T(r) dr$ means the eddy diffusivity of heat.

The concrete form of Green's functions V and W is determined by a geometry considered.

In the simplest case when the dependence of the coefficients Eqs (8) and (20) upon the coordinates can be assumed negligible, Green's function may be found through the use of Fourier transformation.

Then Green's function for the velocity is given by

$$V(\bar{r} - \bar{r}^1, t - t^1) = \frac{1}{2\sqrt{\pi(\nu + D)(t - t^1)^3}} e^{-\frac{|\bar{r} - \bar{r}^1 - U(t - t^1)|^2}{4(\nu + D)(t - t^1)}} \quad (24)$$

Green's function for the temperature W is the same, but we must use respectively a^T , D^T instead of ν , D .

The approximation described above can be used to establish some semi-empirical methods of the calculation of the turbulent flow characteristics. However, in practice it is convenient to carry out the calculations using a more rough scheme.

Consider fully developed flow in a circular tube. The equation for liquid motion is

$$\frac{1}{\xi} \frac{d}{d\xi} \xi (\nu + \nu_T) \frac{dU}{d\xi} = \frac{R^2}{\rho} \cdot \frac{d\rho}{dz} = -2 \frac{r_w}{\rho} R \quad (25)$$

With the eddy diffusivity considered as a known function of the coordinates, after double integration we obtain

$$U(\xi) = \frac{r_w R}{\rho \nu} \int \frac{\xi d\xi}{\xi + \frac{\nu_T}{\nu}} \quad (26)$$

Using the common notations

$$V_*^2 = \frac{r_w}{\rho} , \quad \frac{V_* R}{\nu} = \eta_0 , \quad \xi = \frac{2}{R}$$

we define

$$\begin{aligned} \phi(\xi) &= \frac{U(\xi)}{\eta_0} = \eta_0 \int \frac{\xi d\xi}{\xi + \frac{\nu_T}{\nu}} \\ w(\xi) &= \frac{U(\xi)}{\bar{U}} = \frac{\int \frac{\xi d\xi}{\nu_T}}{\int_0^1 \frac{\xi^3 d\xi}{\nu_T}} \quad (27) \end{aligned}$$

$$Re = \frac{2 \bar{U} R}{\nu} = 2 \eta_0^2 \int_0^1 \frac{\xi^3 d\xi}{1 + \frac{\nu_T}{\nu}}$$

The friction factor ζ is

$$\zeta = \frac{r_w}{\rho \bar{U}^2} = \frac{8}{(\frac{\bar{U}}{V_*})^2} = F(Re) \quad (28)$$

The function $\frac{\nu_T}{\nu} = f(\xi, \eta_0)$ being known, all flow characteristics may be calculated. A good expression for ν_T has been proposed by Reihardt [3]

$$\frac{\nu_T}{\nu} = 0.4 (\eta - 11 \operatorname{th} \frac{\eta}{11}), \quad \eta = \frac{\nu_* y}{\nu} = \eta_0 (1 - \xi) \quad (29)$$

According to this formula the eddy diffusivity of momentum ν_T varies in the viscous layer as $\frac{\nu_T}{\nu} \sim y^3$ and for $\eta \gg 11$ $\frac{\nu_T}{\nu} \sim 0.4\eta$. Since in the turbulent core the velocity profile is described by the relation

$$\phi = 2.5 \ln \eta + 5.5 \quad (30)$$

the eddy diffusivity is to vary in the flow core as

$$\frac{\nu_T}{\nu} = \frac{1 - \frac{\eta}{\eta_0}}{\frac{d\phi}{d\eta}} = \frac{\xi}{\frac{d\phi}{d\eta}} = 0.4 \eta \xi \quad (31)$$

So it is reasonable to correct Reihardt's relation and to determine the ratio of eddy and molecular viscosity as

$$\frac{\nu_T}{\nu} = 0.4 [\eta - 11 \operatorname{th} \frac{\eta}{11}] \xi = 0.4 [\eta_0 (1 - \xi) - 11 \operatorname{th} \frac{\eta_0 (1 - \xi)}{11}] \xi \quad (32)$$

The results of the velocity profile calculations based on Eqs (27) and (32) are given in Fig.1. In Fig.2 the friction factor ζ is plotted as a function of the number Re .

The stabilized heat transfer coefficient calculation reduces to computation of the well-known Lyon integral

$$\frac{1}{Nu} = 2 \int_0^1 \frac{d\xi}{\xi (1 + \frac{a_T}{a})} \left(\int_0^\xi W(\xi) \xi d\xi \right)^2 \quad (33)$$

$$\frac{a_T}{a} = \frac{a_T}{\nu_T} \cdot \frac{\nu_T}{\nu} \cdot \frac{\nu}{a} \approx Pr \cdot \frac{\nu_T}{\nu}$$

Assuming the turbulent Prandtl number equals to unity and using relation (32), the Nusselt number has been calculated in the range $0 < Pr \leq 100$, $5 \cdot 10^3 < Re < 10^6$. The results of the calculations are presented in Figs 3 and 4.

In the figures these results are compared with data from the existing approximate equations. It is seen from this comparison that the approximation of the turbulent viscosity by relation (32) is satisfactory.

3. LOCAL HEAT TRANSFER IN A REACTOR CHANNEL

As known, the heat transfer coefficient is dependent poorly upon a kind of the boundary conditions. However, in the nuclear reactors, where heat generation changes essentially it is necessary to take into account the effect of the axial heat flux variations on the heat transfer coefficient.

Consider the heat transfer process in a circular tube. Let the heat flux $q(z)$ change arbitrarily along the channel. What heat flux variation rate can result in local heat transfer coefficient and local temperature difference changes?

Consider at first the ideal case of the δ -wise heat flux, i.e., the case when heat is generated in an infinitely small element of the channel length. How the temperature profile will be changed down the coolant flow?

Obviously, in the heat source influence area the liquid near-wall layers temperature is much more than the flow core temperature. However, in time the liquid layers are mixing in the channel and the temperature profile is flattening. The length at which temperature profile flattening occurs characterizes the temperature profile development rate. Let the function describing the temperature difference change along the channel is a heat flux influence function and note it as $G(z)$.

Define the local temperature difference as

$$(T_w - \bar{T})c\gamma \bar{U} = \frac{\pi}{2} \int_{-\infty}^z q(z^1)G(z - z^1)dz^1 \quad (34)$$

or

$$\frac{1}{St} = \frac{T_w - \bar{T}}{q(x)} = \frac{\int_{-\infty}^x f(x^1) G(x - x^1) dx^1}{c\gamma \bar{U}} \quad (35)$$

where

$$f(x^1) = \frac{q(x^1)}{q(x)}, \quad x = \frac{z}{R}.$$

If the heat flux is δ -wise, i.e., $f(x) = \delta(x)$, then

$$\frac{(T_w - \bar{T})}{q} \cdot c\gamma \bar{U} = G(x)$$

The function $G(x)$ can be normalized so that $\int_0^\infty G(x)dx = A$. It is characterized by the relaxation length $L = \frac{1}{A} \int_0^\infty x G(x)dx$ and also by the high space moments $\bar{x^n} = \frac{1}{A} \int_0^\infty x^n G(x)dx$.

The influence function moments can be found by Fourier transformation.

Let

$$\tilde{G}(p) = \int_0^\infty e^{-px} G(x) dx \approx \int_0^\infty G(x) dx - p \int_0^\infty x G(x) dx + \frac{p^2}{2} \int_0^\infty x^2 G(x) dx - \dots = A[1 - pL + \frac{p^2}{2} \bar{x}^2 - \dots] \quad (36)$$

Then

$$A = \int_0^\infty G(x) dx = \tilde{G}(p)|_{p \rightarrow 0} \quad (37)$$

$$L = \frac{1}{A} \frac{d \tilde{G}(p)}{dp}|_{p \rightarrow 0}, \bar{x}^2 = \frac{1}{A} \frac{d^2 \tilde{G}}{dp^2}|_{p \rightarrow 0}, \text{etc.}$$

If a function $f(x)$ changes poorly at distances of the order of L , it can be expanded into a Taylor series and then

$$\frac{1}{St(x)} = \frac{T_w - \bar{T}}{\frac{q(x)}{c \gamma \bar{U}}} = \frac{x}{-\infty} \int f(x^1) G(x - x^1) dx^1 = \int_0^\infty f(x - x^1) G(x^1) dx^1 \approx f(x) \int_0^\infty G(x) dx - \frac{df}{dx} \int_0^\infty x G(x) dx + \dots = A[f(x) - L \frac{df}{dx} + \dots] \approx A f(x - L) \quad (38)$$

Thus the local heat transfer coefficient is determined by the heat flux at the distance L up the flow.

As we will see later, the factor A is the inverse Stanton number after full development of the temperature profile.

In this connection

$$\frac{1}{St(x)} \approx \frac{1}{St_0} f(x - L) \quad (39)$$

Thus using the stabilized heat transfer coefficient, we underestimate the heat transfer coefficient value when heat flux increases and overestimate it when heat flux decreases along a channel*. The last circumstance is essential for the water cooled reactors in which subcooled boiling is possible at core outlet. Therefore the fuel element surface temperature estimations and analysis of the subcooled boiling possibility performed in accordance with the usual heat transfer relations may be appeared quite optimistic.

Surely, correctness of such estimations depends upon the rate of heat flux change along a channel. If in expansion (38) the term $L \frac{df}{dx}$ can be neglected, i.e., $L \frac{d \ln f(x)}{dx} \ll 1$, then the heat flux nonuniformity effect on heat transfer coefficient is small. Otherwise, if relative heat flux variations are small at distances about the relaxation length L , then this effect can be ignored.

It is required to know the influence function $G(x)$ to calculate the local heat transfer coefficient for varying heat flux along a channel. The influence function $G(x)$ can be determined from the solution of the energy equation.

* It may be shown such a situation takes place in transient processes also, when $q = \text{const} \cdot f(t)$

This equation for liquid flowing in a circular tube is

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \left(1 + \frac{aT}{a}\right) \frac{\partial T}{\partial \xi} = \frac{U(\xi)}{a} R \frac{\partial T}{\partial x} = W(\xi) \frac{Pe}{2} \frac{\partial T}{\partial x} \quad (40)$$

and the boundary condition is

$$\left. \frac{dT}{d\xi} \right|_1 = \frac{q(x)}{\lambda} R \quad (41)$$

The solution Eq. (40) can be determined by means of the finite integral transformation over the eigenfunctions of Sturm-Liouville's equation

$$\frac{1}{\xi} \frac{d}{d\xi} \xi \left(1 + \frac{aT}{a}\right) \frac{d Y_n}{d\xi} + \beta_n^2 w(\xi) Y_n = 0 \quad (42)$$

with the boundary condition $\left. \frac{d Y_n}{d\xi} \right|_1 = 0$. With the more general boundary condition

$\left. \frac{d Y_n}{d\xi} \right|_1 = z Y_n$ (1) more complex problems may be solved.

Using the orthogonality of the function Y_n , i.e., the equalities $\int_0^1 \xi w(\xi) Y_n Y_m d\xi = 0$ with $n \neq m$, any function given in the interval [0,1] may be expanded into the series

$$\theta(\xi, x) = \sum_n \frac{Y_n(\xi)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} \int_0^1 \xi w(\xi) \theta(\xi, x) Y_n(\xi) d\xi \quad (43)$$

If to denote

$$\tilde{\theta}_n(\xi) \approx \int_0^1 \xi w(\xi) \theta(\xi, x) Y_n(\xi) d\xi, \quad (44)$$

then the equality

$$\theta(\xi, x) = \sum_n \frac{Y_n(\xi) \tilde{\theta}_n(x)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} \quad (45)$$

may be considered as the conversing formula for the finite integral transformation.

Applying the integral transformation in the form of (44) to Eq. (40), i.e. multiplying the equation by the function $Y_n(\xi)$ and then integrating it from zero to unity, we obtain the equation

$$\frac{Pe}{2} \frac{dT_n}{dx} = Y_n(1) \frac{q(x) R}{\lambda} - \beta_n^2 \tilde{T}_n(x), \quad (46)$$

the solution of which is

$$\tilde{T}_n(x) = Y_n(1) \frac{2R}{\lambda Pe} \int_{-\infty}^x q(x^1) e^{-\frac{2}{Pe} \beta_n^2 (x - x^1)} dx^1, \frac{2R}{\lambda Pe} = \frac{1}{c \gamma U}. \quad (47)$$

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In accordance with Eq. (45) the temperature profile in the channel is determined by the series

$$T(\xi, x) = \frac{1}{cy \bar{U}_n} \sum_n \frac{Y_n(\xi) Y_n(1)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} \int_{-\infty}^x q(x^1) e^{-\frac{2}{Pe} \beta_n^2 (x-x^1)} dx^1 \quad (48)$$

Near the surface the temperature difference is

$$T_w - \bar{T} = \frac{1}{cy \bar{U}} \sum_{n \neq 0} \frac{Y_n^2(1)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} \int_{-\infty}^x q(x^1) e^{-\frac{2}{Pe} \beta_n^2 (x-x^1)} dx^1, \quad (49)$$

and the inverse Stanton number

$$\frac{1}{St(x)} = \frac{T_w - \bar{T}}{q(x)} = \frac{x}{\int_{-\infty}^x} \left[\sum_{n \neq 0} \frac{Y_n^2(1)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} e^{-\frac{2}{Pe} \beta_n^2 (x-x^1)} \right] f(x^1) dx^1 \quad (50)$$

Thus the heat flux influence function introduced before is

$$G(x) = \sum_{n \neq 0} \frac{Y_n^2(1)}{\int_0^1 \xi w(\xi) Y_n^2 d\xi} e^{-\frac{2}{Pe} \beta_n^2 x} \quad (51)$$

If the heat flux is constant, then the local Stanton number is

$$\frac{1}{St(x)} = \frac{Pe}{2} \sum_{n \neq 0} \frac{Y_n^2(1)}{\beta_n^2 \int_0^1 \xi w(\xi) Y_n^2 d\xi} [1 - e^{-\frac{2}{Pe} \beta_n^2 x}] \quad (52)$$

With $x \rightarrow \infty$ heat transfer is stabilized and

$$\frac{1}{St(x)} \Big|_{x \rightarrow \infty} = \frac{1}{St_0} = \frac{Pe}{2} \sum_{n \neq 0} \frac{Y_n^2(1)}{\beta_n^2 \int_0^1 \xi w(\xi) Y_n^2 d\xi} \quad (53)$$

Calculating the moments of the function $G(x)$ from Eq. (37) it can be shown that

$$A = \int_0^\infty G(x) dx = \frac{Pe}{2} \sum_{n \neq 0} \frac{Y_n^2(1)}{\beta_n^2 \int_0^1 \xi w(\xi) Y_n^2 d\xi} = \frac{1}{St_0} \quad (54)$$

$$L = St_0 \int_0^\infty x G(x) dx = \frac{Pe}{2} \frac{\sum_{n \neq 0} \frac{Y_n^2(1)}{\beta_n^4 \int_0^1 \xi w(\xi) Y_n^2 d\xi}}{\sum_{n \neq 0} \frac{1}{\beta_n^2 \int_0^1 \xi w(\xi) Y_n^2 d\xi}} \quad (54)$$

Limiting expansion (36) by the first two terms, i.e.,

$$\tilde{G}(p) \approx \frac{1}{St_0} (1 - pL) \approx \frac{1}{St_0} \cdot \frac{1}{1+pL} \quad (55)$$

we have the approximate representation of the function $G(x)$ in the form

$$G(x) = \frac{1}{St_0} \frac{1}{L} e^{-\frac{x}{L}} \quad (56)$$

It permits to calculate the local Stanton number for arbitrary heat flux distribution through the use of the relation

$$\frac{1}{St(x)} = \frac{1}{St_0} \frac{1}{L} \int_{-\infty}^x f(x^1) e^{-\frac{x-x^1}{L}} dx^1 \quad (57)$$

In particular for $q = \text{const}$ in the thermal stabilization region

$$\frac{1}{St(x)} = \frac{1}{St_0} (1 - e^{-\frac{x}{L}}) \quad (58)$$

The data on the heat transfer in the entrance section of tubes should be presented just in such a form. The thermal stabilization length may be considered as trebled relaxation length. It is corresponding to that $\frac{St}{St_0}$ differs from unity by 5 per cent

For practical calculations of the local heat transfer coefficient in case of complex heat flux distribution it is required to know the system of the eigenfunctions $Y_n(\xi)$. The calculations such the functions and their eigenvalues have been performed using the code developed by A.I.Kuleshov and E.D.Beljaeva for computer M-20. The eddy and molecular diffusivity of heat ratio $\frac{aT}{a} \approx \text{Pr} \cdot \frac{\nu T}{\nu}$ have been calculated using Eq. (32), and the velocity profile $w(\xi)$ have been calculated using Eq. (27). The results of the thermal stabilization length calculations are given in Table 1.

Thermal stabilization length values $\frac{1}{d} = \frac{3}{2} L$

Table 1

Re	Pr = 0.01			Pr = 0.025		Pr = 1.0
	10^4	$5 \cdot 10^4$	10^5	10^4	10^5	10^4
$\frac{1}{d}$	2.43	8.5	12.8	5.01	14.2	13.7

In Fig. 5 Subbotin's experimental data [7] on heat transfer for liquid metals in the entrance section of tubes are compared with the results of the calculations. The thermal stabilization length obtained from the experiments is in good agreement with the calculation.

The local Nusselt number is presented in Fig. 6 for the sinusoidal heat flux. It is seen that for the decreasing part of the sinusoid the number Nu is 20-40% less than the stabilized value.

4. FRICTION AND HEAT TRANSFER IN PARALLEL FLOW THROUGH BUNDLES OF RODS

Fuel assemblies of nuclear reactors are often made as bundles of cylindrical rods. Calculations of friction and heat transfer in such channels are usually performed with the use of the relations obtained for flow in tubes. It is considered that the equivalent diameter defined as

$d_h = \frac{4S}{\pi}$ is adequate to account for a channel geometry. However, it has been shown in some experimental studies [8] that the equivalent diameter is not the universal parameter and does not provide simple describing of experimental data for bundles of rods.

Therefore it is reasonable to carry out theoretical calculations of friction and heat transfer in such a geometry.

For turbulent flow the full solution of the motion and energy equations can be obtained only numerically in such complex geometry. Then some difficulties arise associated with the turbulent flow characteristics uncertainty. Therefore it is advisable to perform the calculation approximately basing, however, on a reasonable description of turbulent flow properties.

By such an approximate calculational method may be well-known in neutron physics Wigner-Seitz method in which a real lattice cell is replaced by some symmetric cylindrical one. It is obvious that such an operation can be applied only to extend bundles when it may be neglected by the azimuthal variations of shear stresses and the temperature difference over a rod perimeter. In such a symmetrized geometry heat transfer calculations have been made for liquid metal flow by Dwyer and Tee and also by Fridlander and Bonilla [9].

a) Friction factor of a bundle of rods.

Consider coolant flow in an annulus formed between the rod surface and the cell boundary.

After integrating twice of the motion equation (25) with using of the boundary conditions we will have the velocity distribution over the annulus of the form.

$$U(\xi) = - \frac{R^2}{2\mu} \cdot \frac{dp}{dz} \int \frac{\xi}{\xi_0} \frac{d\xi (1 - \xi^2)}{\xi (1 + \frac{\nu T}{\nu})} \quad (59)$$

where $\xi_0 = \frac{\rho}{R}$.

The mean velocity over the annulus

$$\bar{U} = - \frac{R^2 \frac{dp}{dz}}{2\mu(1 - \xi_0^2)} \int_{\xi_0}^1 \frac{(1 - \xi^2)^2 d\xi}{\xi(1 + \frac{\nu T}{\nu})} \quad (60)$$

The eddy viscosity given - the velocity profile and friction factor can be calculated.

$$\zeta = \frac{8 r_w}{\bar{U} \cdot \frac{\gamma}{g}} = \frac{8}{\bar{\phi}^2}, \quad (61)$$

where

$$\bar{\phi} = \frac{\bar{U}}{v_*} = \frac{\eta_0}{(1 - \xi_0)} \cdot \frac{\xi_0}{(1 - \xi_0^2)^2} \int_{\xi_0}^1 \frac{(1 - \xi^2)^2 d\xi}{\xi(1 + \frac{\nu T}{\nu})}.$$

The quantity $\eta_0 = \frac{v_*(R - \rho)}{\nu}$ is the characteristic parameter for turbulent flow in the annulus. The Reynolds number calculated through the equivalent diameter is determined by the relation

$$Re = \frac{\bar{U} d_h}{\nu} = \frac{2\eta_0^2}{(1 - \xi_0^2)(1 - \xi_0)^2} \int_{\xi_0}^1 \frac{(1 - \xi^2)^2 d\xi}{\xi(1 + \frac{\nu T}{\nu})} \quad (62)$$

For laminar flow along a bundle of rods it can be made exact friction calculations. But these calculations are very complicated. Using Eqs (60), (61) and assuming $\nu T = 0$

$$-\frac{dp}{dz} = \frac{\bar{U}}{\frac{R^2}{4\mu} \left(\frac{\epsilon}{2} - \frac{3}{2} - \frac{\ln \epsilon}{1-\epsilon} \right)} = \frac{\gamma \bar{U}^2}{2g} \cdot \frac{1}{\frac{\bar{U} \gamma}{g \mu} \cdot \frac{R^2}{8} \left(\frac{\epsilon}{2} - \frac{3}{2} - \frac{\ln \epsilon}{1-\epsilon} \right)} = \zeta \frac{\gamma \bar{U}^2}{2g d_h} \quad (63)$$

where

$$\zeta = \frac{64}{\frac{\bar{U} \gamma}{g \mu} d_h x}, \quad x = \frac{2\epsilon}{(1-\epsilon)^2} \left(\frac{\epsilon}{2} - \frac{3}{2} - \frac{\ln \epsilon}{1-\epsilon} \right), \quad (64)$$

$$\epsilon = \frac{\pi \rho^2}{\pi R^2} = \xi_0^2$$

Let the quantity $d_h \cdot x = d_e$ be the effective diameter. With such a characteristic diameter definition the bundle friction law coincides with friction law for tubes, i.e.

$$\zeta = \frac{64}{Re \cdot x} = \frac{64}{Re_e} \quad (65)$$

When $\epsilon \rightarrow 1$, $x \rightarrow \frac{2}{3}$ and $\zeta \rightarrow \frac{96}{Re}$ as for a plane flat duct.

It is required to know the eddy viscosity variation law over a channel section to calculate the friction factor of a bundle of rods. It may be assumed that the eddy viscosity distribution for a bundle of rods is described by the relation of the form

$$\frac{\nu T}{\nu} = 0.4 \left[\eta_0 \frac{\xi - \xi_0}{1 - \xi_0} - 11 \operatorname{th} \frac{\eta_0 (\xi - \xi_0)}{11(1 - \xi_0)} \right] \frac{1 - \xi}{1 - \xi_0} \quad (66)$$

Using relations (61) and (63) it may be calculated the friction factor ζ as a function of the Reynolds number and the lattice density ϵ . The results of the calculations are shown in Fig. 7.

It may be seen that the coefficient ζ is not a simple function of the number Re , as it has been obtained from experiments.

If the Reynolds number is defined through the effective diameter

$$Re_e = Re \cdot x \quad (67)$$

then the exfoliation of the dependence $\zeta = f(Re)$ with respect to the parameter ϵ decreases essentially (Fig. 7). These results permit to conclude that the effective diameter is a better geometrical characteristic of a bundle of rods than the equivalent diameter. However, it is true of the wide bundles only.

b) The heat transfer coefficient for parallel flow through a bundle of rods.

The equivalent cell method may be also used to calculate heat transfer in bundles of rods. The energy equation (40) integrating with the corresponding boundary conditions over the annulus, the expression for the Nusselt number will be found

$$\frac{1}{Nu} = \frac{2\xi_0^2}{(1-\xi_0^2)^3} \frac{1}{\xi_0} \frac{d\xi}{\xi(1+Pr \frac{\nu T}{\nu})} \left(\frac{1}{\xi} \xi w(\xi) d\xi \right)^2 \quad (68)$$

Heat transfer calculations for a bundle of rods are performed using the velocity profile determined Eq. (59) and the eddy viscosity Eq. (66).

The results of the calculations are presented in Figs 8,9,10. The Nusselt number calculated through the equivalent diameter is not a simple function of the Reynolds number as is the friction factor ζ .

If the effective diameter is introduced, then the exfoliation of the dependences

$$Nu = f(Re), \quad St = f(Re)$$

decreases essentially. For this the data calculated are scattered by 10-15% from the criterial relationship obtained for ducts. This result shows also that processing of data on heat transfer in bundles of rods should be made using the effective diameter and not the equivalent diameter.

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NOMENCLATURE

a	— molecular diffusivity of heat
p	— pressure
Π	— heat transfer perimeter
R	— tube radius, cell radius
$u(r, t) = U + u^1$	— flow velocity
$T(r, t) = T + T^1$	— temperature
S	— cross-sectional flow area
t, r	— time
x, y, z	— coordinates
γ	— specific weight
$\delta(\bar{r})$	— delta-function
μ	— viscosity
ρ	— liquid density, radius of rod
ν	— kinematic viscosity
ξ	— dimensionless radial coordinate
$\epsilon = \left(\frac{\rho}{R}\right)^2$	— "density" of lattice of rods
$\eta = \frac{v_* y}{\nu}$	— dimensionless distance from the wall
ζ	— friction factor
τ	— shear stress
Nu, Re, St, Pe, Pr	— Nusselt, Reynolds, Stanton, Pecl, Prandtl numbers

INDEXES

- w — wall
 T — turbulent
 o — stabilized value
 1 — pulsational component
 — (dash) — notation of averaging
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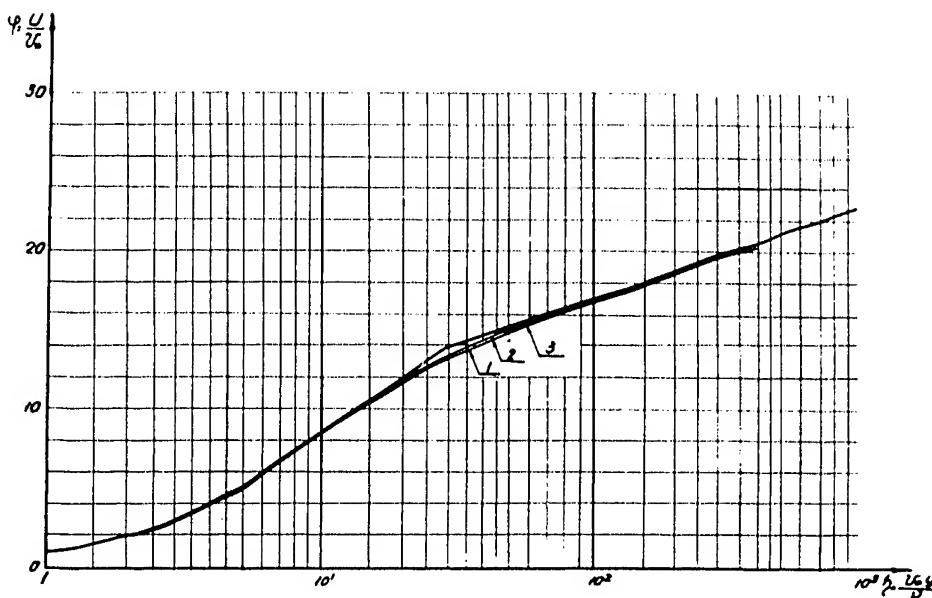
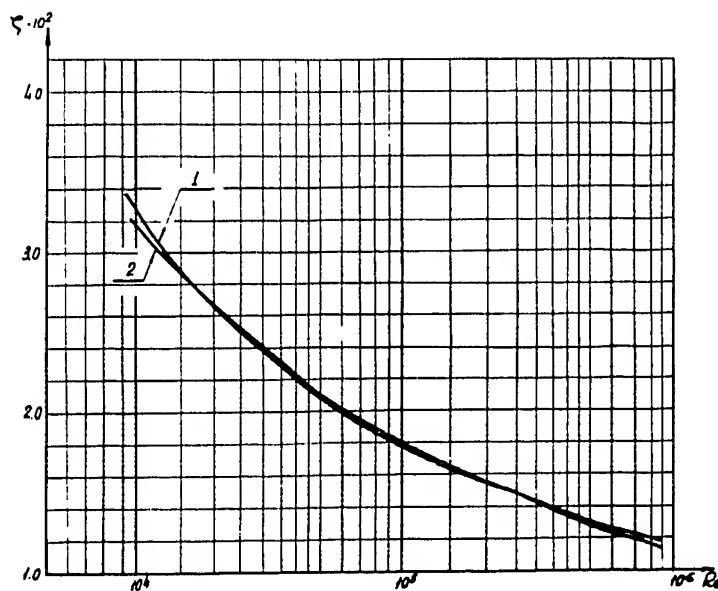


FIG.1. LIQUID VELOCITY PROFILE IN A TUBE

1 — $Re = 10^4$; 2 — $Re = 10^5$; 3 — Approximation: $\phi = \eta$ $\eta < 5$
 $\phi = 5 \ln \eta - 3.05$ $5 < \eta < 30$ $\phi = 2.5 \ln \eta + 5.5$ $\eta > 30$

FIG.2. FRICTION FACTOR ζ AS A FUNCTION OF THE REYNOLDS NUMBER IN A TUBE

1 — Calculation using Eqs (27, 28, 32); 2 — $\zeta = \frac{0.3164}{Re^{0.25}}$ and

$$326 \quad \zeta = \frac{1}{(1.82 \lg Re - 1.64)^2}$$

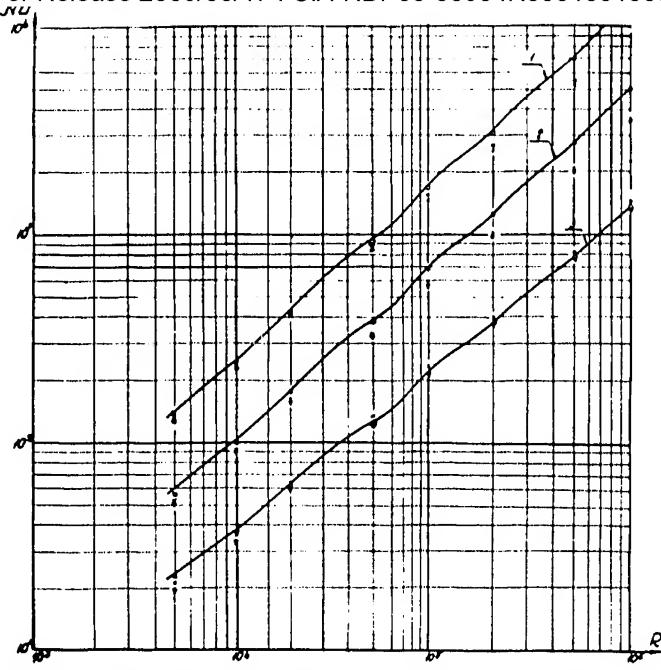


FIG.3. FUNCTION $\text{Nu} = f(\text{Re}, \text{Pr})$ WITH $\text{Pr} > 1$ FOR
LIQUID FLOW IN TUBES

1 - $\text{Pr} = 100$; 2 - $\text{Pr} = 10$; 3 - $\text{Pr} = 1.0$; $\times \text{Nu} = 0.023$
 $\text{Re}^{0.8} \text{Pr}^{0.4}$ [4]; $\nabla \text{Nu} = 0.021 \text{Re}^{0.8} \text{Pr}^{0.43}$ [5]

$$\circ \text{Nu} = \frac{\text{Re} \cdot \text{Pr}}{8 \left[\frac{4.5(\text{Pr}^{2/3} - 1)}{1.82 \lg \text{Re} - 1.64} + 1.07 \right] (1.82 \lg \text{Re} - 1.64)^2} \quad [6]$$

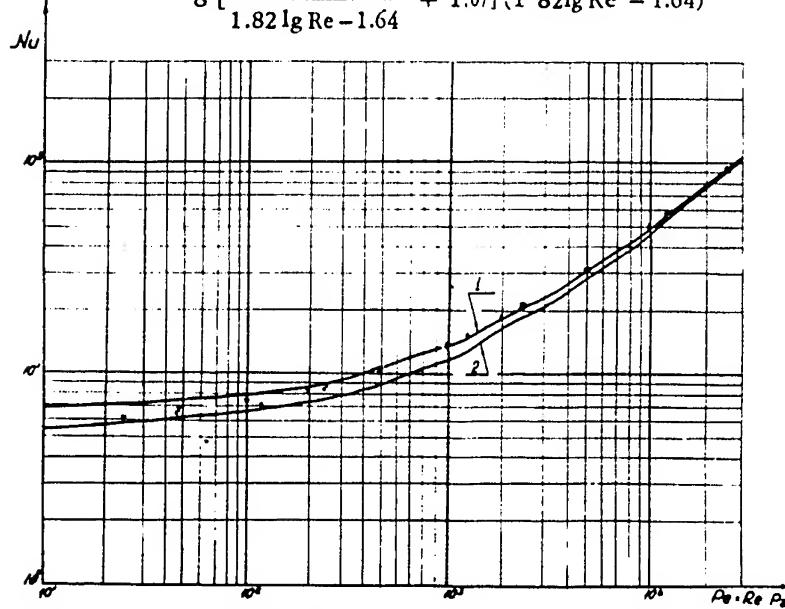


FIG.4. FUNCTION $\text{Nu} = f(\text{Pe})$ FOR LIQUID METAL FLOW IN
TUBES

1 - $\text{Nu} = 7 + 0.025 \text{Pe}^{0.8}$; 2. $\text{Nu} = 5.5 + 0.025 \text{Pe}^{0.8}$; $\nabla \text{Pr} = 0.005$;
 $\times \text{Pr} = 0.01$; $\circ \text{Pr} = 0.025$; $\Delta \text{Pr} = 0.05$

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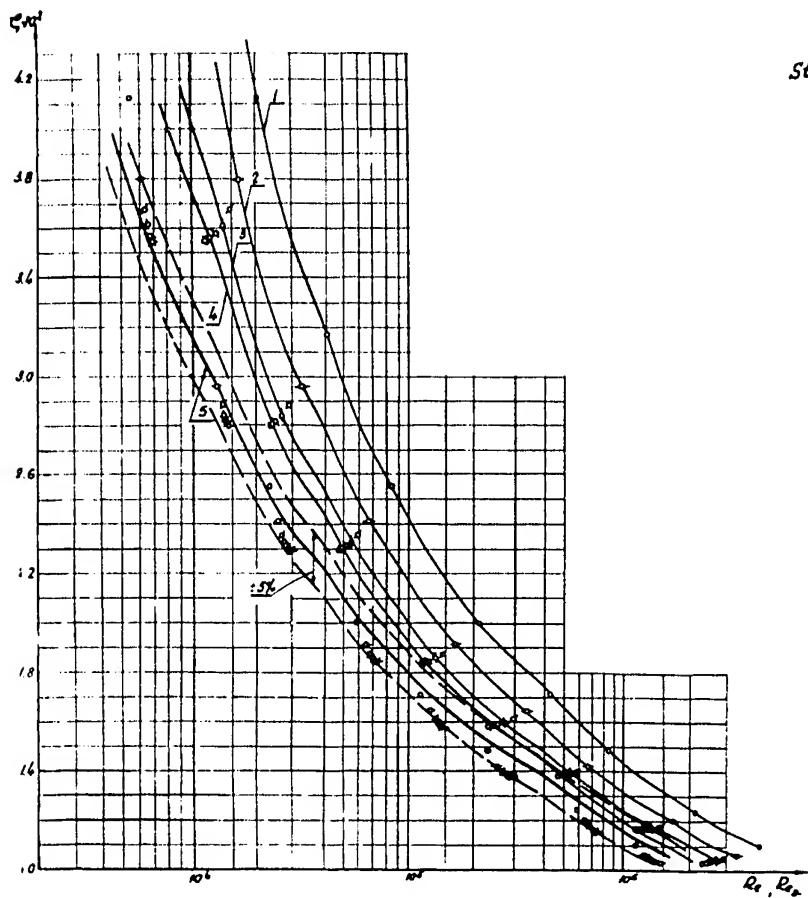


FIG.7. FUNCTIONS $\zeta = f(Re)$ AND $\zeta = f(Re_e)$ FOR A BUNDLE OF RODS

1. $\circ - \epsilon = 0.1$

3. $\times - \epsilon = 0.4$

2. $-o- - \epsilon = 0.2$

$\times - \epsilon = 0.5$

$\phi - \epsilon = 0.3$

$\circ - \epsilon = 0.6$

5. $\zeta = \frac{1}{(1.82 \lg Re - 1.64)^2}$

Points at left relate to $Re_e = Re \cdot f$

$$Re = \frac{\bar{U} d_h}{\nu}, \quad x = \frac{2\epsilon}{(1-\epsilon)^2} \left[\frac{\epsilon}{2} - \frac{3}{2} - \frac{\ln \epsilon}{1-\epsilon} \right], \quad \epsilon = \frac{\rho^2}{R^2}$$

ρ - rod radius, R - cell radius

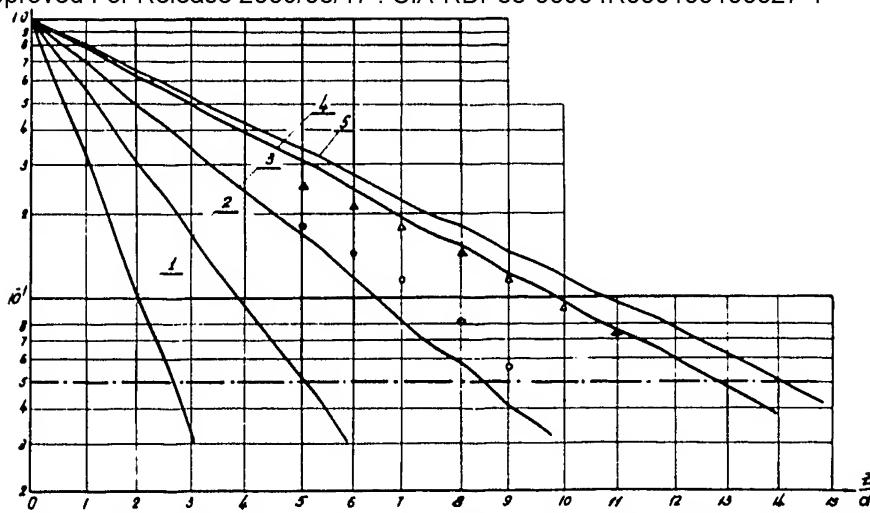


FIG.5. COMPARISON OF CALCULATIONAL AND EXPERIMENTAL DATA
ON HEAT TRANSFER FOR LIQUID METAL FLOW IN THE ENTRANCE SEC-
TION OF TUBES

Experiment [7] o $Pe = 500$; $\Delta Pe = 1000$; Calculation: 1 — $Re = 9.67 \cdot 10^3$
 $Pr = 0.01$; 2 — $Re = 9.67 \cdot 10^3$ $Pr = 0.025$; 3 — $Re = 5 \cdot 10^4$ $Pr = 0.01$;
4 — $Re = 10^5$ $Pr = 0.01$; 5 — $Re = 10^5$; $Pr = 0.025$;

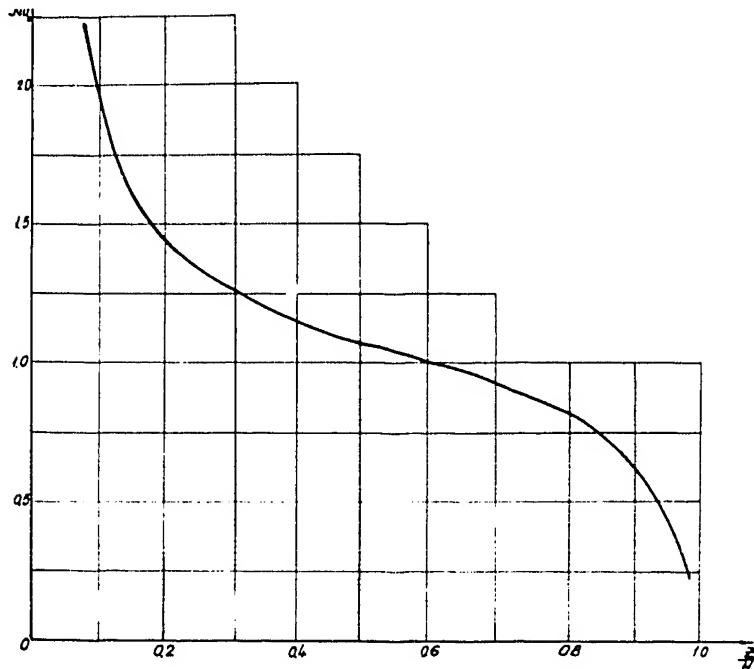


FIG.6. $\frac{Nu}{Nu_0}$ AS A FUNCTION OF RELATIVE CHANNEL LENGTH FOR
SINUSOIDAL HEAT FLUX. $Re = 10^4$, $Pr = 1.0$, $\frac{H}{d} = 50$

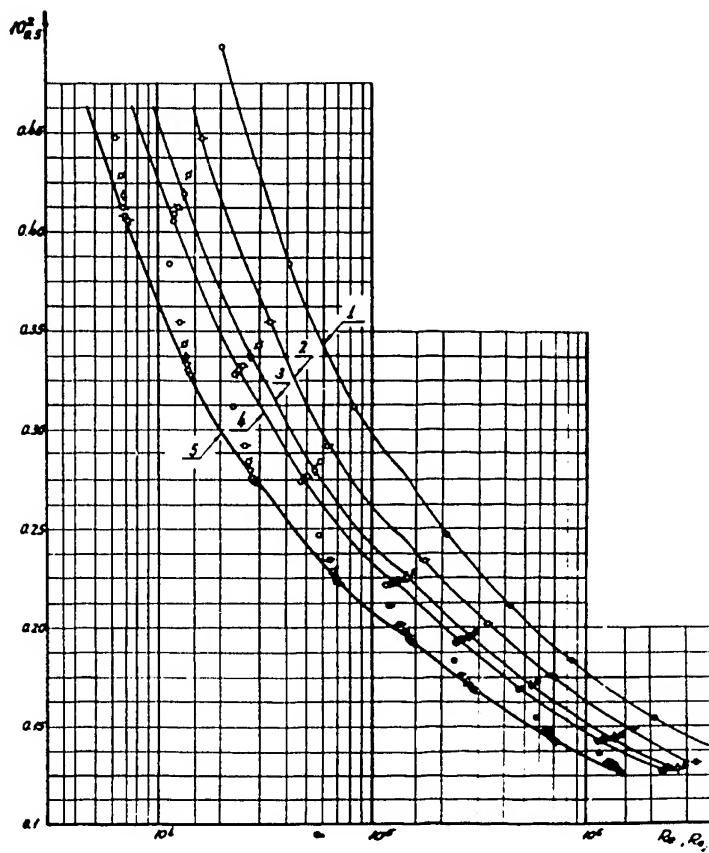


FIG.8. HEAT TRANSFER CALCULATION FOR PARALLEL FLOW THROUGH A BUNDLE OF RODS FOR $Pr = 1.0$

- | | |
|-----------------------------|--------------------------------|
| 1. o - $\epsilon = 0.1$ | 3. $\diamond - \epsilon = 0.4$ |
| 2. -o- - $\epsilon = 0.2$ | 4. $\times - \epsilon = 0.5$ |
| $\diamond - \epsilon = 0.3$ | 5. $\circ - \epsilon = 0.6$ |
| | 6. $\times - \epsilon = 0.7$ |

$$5. St = \frac{\zeta/8}{12.7 \sqrt{\frac{\zeta}{8}} (Pr^{2/3} - 1) + 1.07}$$

$$\zeta = \frac{1}{(1.82 \lg Re - 1.64)^2}$$

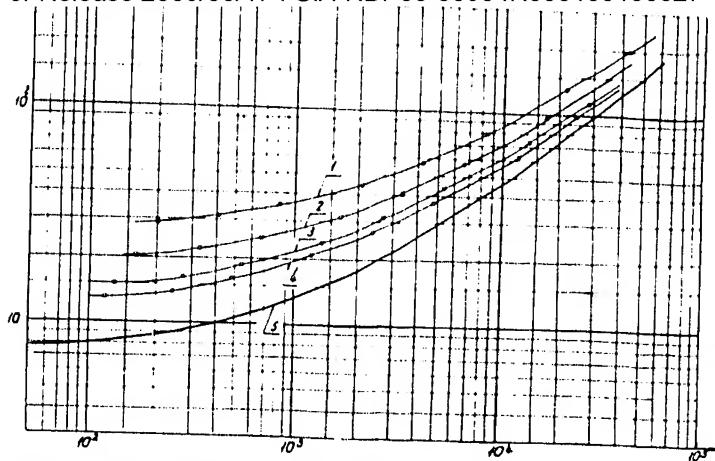


FIG.9. FUNCTION $Nu = f(Pe)$ WITH $Pr = 0.01$ FOR PARALLEL LIQUID METAL FLOW THROUGH A BUNDLE OF RODS

1. $\epsilon = 0.1$
 2. $\epsilon = 0.2$
 3. $\epsilon = 0.4$

4. $\epsilon = 0.6$
 5. $Nu = 7 + 0.025 Pe^{0.8}$

Numbers Nu and Pe are calculated through equivalent diameter

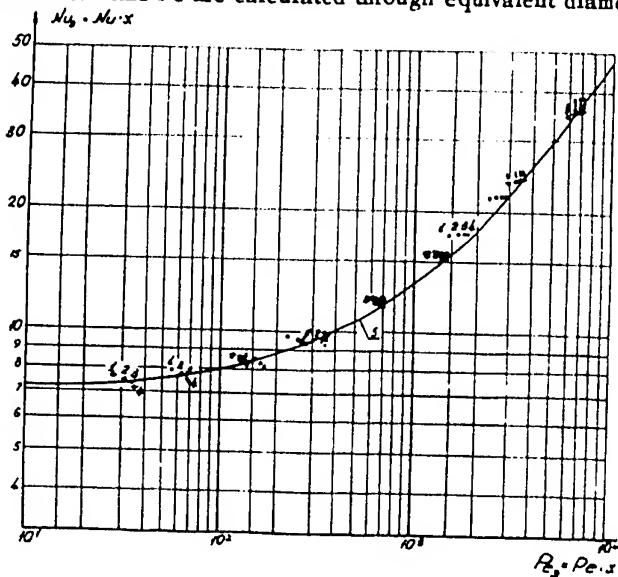


FIG.10. FUNCTION $Nu_e = f(Re_e)$ FOR PARALLEL LIQUID METAL FLOW OVER A BUNDLE OF RODS

- o— $Pr = 0.005$ o $Pr = 0.01$ Δ $Pr = 0.025$
 x $Pr = 0.05$ 1. $\epsilon = 0.1$ 2. $\epsilon = 0.2$ 3. $\epsilon = 0.4$
 4. $\epsilon = 0.6$ 5. $Nu_e = 7 + 0.025 Pe_e^{0.8}$ $Nu_e = Nu \cdot f$

$$Pe_e = Pe \cdot f$$